

Contents

List of Figures	vi
Introduction	x
1 An Introduction to Dynamical Systems	1
1.1 What is a dynamical system?	1
1.1.1 State Vectors	1
1.1.2 The next instant: discrete time	2
1.1.3 The next instant: continuous time	3
1.2 What we want; what we can get	4
2 Preliminary Results	5
2.1 Introduction	5
2.2 Definitions of Stability and Linearized Stability	5
2.3 Semi-cycle Analysis	11
2.4 Criterion for the Asymptotic Stability	12
2.5 Global Asymptotic Stability	14
3 Linear and Nonlinear Dynamical Systems	18
3.1 Linear Dynamical Systems	18
3.1.1 One dimension	18
3.1.2 Two (and more) dimensions	27
3.2 Nonlinear Dynamical Systems	31
3.2.1 Fixed points	31
3.2.2 Stability	31
4 Qualitative behavior of the difference equation $x_{n+1} = Ax_n + \frac{\beta x_n + \gamma x_{n-k}}{Bx_n + Cx_{n-k}}$	33
4.1 Change of variables	34

4.2	Linearization	35
4.3	Local stability	38
4.4	Periodic solutions	40
4.5	Invariant intervals	46
4.6	Semi-cycle analysis	53
4.7	Global stability	53
4.8	Numerical Discussion	56
5	Qualitative behavior of the difference equation $x_{n+1} = Ax_n + \frac{px_n+x_{n-k}}{q+x_{n-k}}$	62
5.1	Equilibrium points	62
5.2	Linearization	64
5.2.1	The linearized equation about the positive equilibrium point	64
5.2.2	The linearized equation about the zero equilibrium point	66
5.3	Local stability	66
5.4	Periodic solutions	70
5.5	Invariant intervals	74
5.6	Global stability	79
5.7	Numerical Discussion	81
A	The MATLAB 6.5 Codes	84
A.1	The difference equation $x_{n+1} = Ax_n + \frac{\beta x_n + \gamma x_{n-k}}{Bx_n + Cx_{n-k}}$	84
A.2	The difference equation $x_{n+1} = Ax_n + \frac{px_n+x_{n-k}}{q+x_{n-k}}$	85
References		87